QMCPy Client for UM-Bridge

Aleksei G. Sorokin

Illinois Institute of Technology (IIT), Department of Applied Mathematics

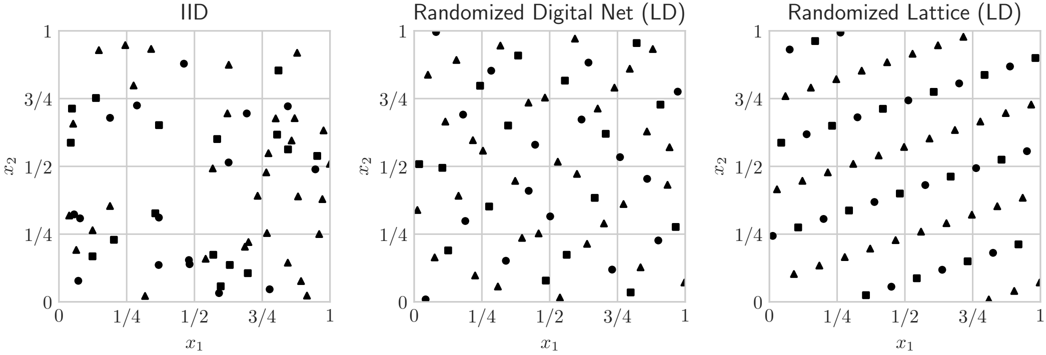
QMCPy: A (Quasi-)Monte Carlo Software in Python

(Q)MC methods efficiently approximate the expectation of a random variable

$$\text{(Exact)} \quad \mu = \mathbb{E}[g(\boldsymbol{T})] = \mathbb{E}[f(\boldsymbol{X})] = \int_{(0,1)^d} f(\boldsymbol{x}) \mathrm{d}\boldsymbol{x} \quad \approx \quad \frac{1}{n}\sum_{i=1}^{n} f(\boldsymbol{x}_i) = \hat{\mu} \quad \text{(Approx)}$$

- want the expectation of g WRT r.v. $\boldsymbol{T}$
  - cantilevered beam displacement WRT uncertain material parameters
  - payoff of a financial option WRT Brownian motion
- transform to equivalent expectation of f WRT $\boldsymbol{X} \sim \mathcal{U}(0,1)^d$
- $\boldsymbol{x}_1, \dots, \boldsymbol{x}_n \sim \mathcal{U}(0,1)^d$ sampling nodes
  - IID for Simple Monte Carlo: $\mathcal{O}(1/\sqrt{n})$ convergence
  - low discrepancy for Quasi-Monte Carlo: nearly $\mathcal{O}(1/n)$ convergence
- approximate the function average by the sample average

IID vs Low Discrepancy Points



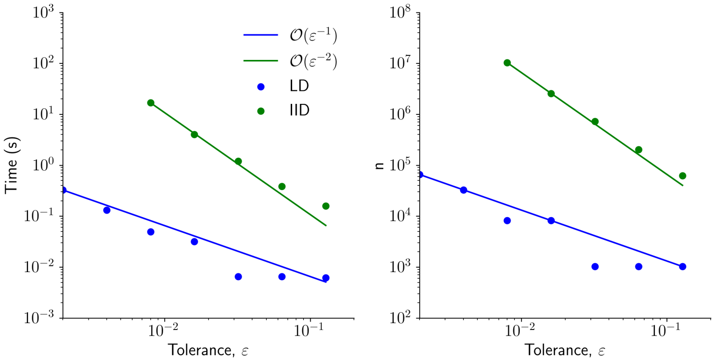
QMCPy Components

$$\text{(Exact)} \quad \mu = \mathbb{E}[g(\boldsymbol{T})] = \mathbb{E}[f(\boldsymbol{X})] = \int_{(0,1)^d} f(\boldsymbol{x}) \mathrm{d}\boldsymbol{x} \quad \approx \quad \frac{1}{n} \sum_{i=1}^{n} f(\boldsymbol{x}_i) = \hat{\mu} \quad \text{(Approx)}$$

- generator of $\boldsymbol{x}_1, \dots, \boldsymbol{x}_n$: **Discrete Distribution**
- transform setting f so $\mathbb{E}[g(\boldsymbol{T})] = \mathbb{E}[f(\boldsymbol{X})]$ where $\boldsymbol{X} \sim \mathcal{U}(0,1)^d$: **True Measure**
- model g: **Integrand**
- stopping criterion algorithm adaptively determining n s.t.

$$\text{approximation error} = |\mu - \hat{\mu}| \leq \varepsilon = \text{user error threshold}$$

Keister Integral: $g(\boldsymbol{T}) = \pi^{d/2} \cos(\|\boldsymbol{T}\|_2)$, $\boldsymbol{T} \sim \mathcal{N}(\boldsymbol{0}, \boldsymbol{I}/2)$



B. D. Keister. “Multidimensional Quadrature Algorithms”. In: *Computers in Physics* 10 (1996), pp. 119–122. DOI: 10.1063/1.168565

References and Code Demo

- QMCPy ReadTheDocs UM-Bridge Demo: https://qmcpy.readthedocs.io/en/latest/demo_rst/umbridge.html
- QMCPy ReadTheDocs `UMBridgeWrapper`: https://qmcpy.readthedocs.io/en/latest/algorithms.html#module-qmcpy.integrand.um_bridge_wrapper
- UM-Bridge ReadTheDocs QMCPy Client : https://um-bridge-benchmarks.readthedocs.io/en/docs/umbridge/clients.html
- QMCPy Homepage: https://qmcpy.org/

- Sou-Cheng T. Choi et al. "Quasi-Monte Carlo Software". In: *Monte Carlo and Quasi-Monte Carlo Methods*. Ed. by Alexander Keller. Cham: Springer International Publishing, 2022, pp. 23–47. ISBN: 978-3-030-98319-2